CHALLENGES IN APPLYING CALIBRATION METHODS TO TRAFFIC MODELS

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ABSTRACT

This text looks at calibration and validation as a means to understand traffic flow models better. It concentrates on the car-following part of it and demonstrates that the calibration of stochastic models under certain circumstances can become very difficult.

Three types of stochasticity are distinguished for microscopic traffic flow models: the one coming from noisy data, the one coming from the distribution of the parameters describing the driver's behavior and the one coming from the model itself, when a noise component is added to a deterministic differential equation governing the vehicle's movements.

By using four sub models comprising four different noise terms and an identical deterministic part this text shows that a calibration with synthetic – and therefore reproducible – data can lead to awry results. Parameters fitted by the calibration procedure are significantly different for deterministic and stochastic models. The text makes the conclusion that the stochasticity is the reason why the parameter estimation of stochastic models fails sometimes. Up to now, the authors were, unfortunately, not able to propose a solution to cope with this intrinsic pitfall of genuine stochastic models.

Keywords: Micro simulation, Car-following models, Trajectory data, Calibration

1 INTRODUCTION

A lot of work has been conducted recently to improve on the usage of microscopic traffic flow models and especially their calibration and validation to real world data, for an overview see [4]. This has worked with an astonishingly degree of quality, i. e. traffic flow models can be calibrated to real data with an r.m.s.-error of the order of around 10 %.

While good enough for daily use, it may have a weak spot, and that is its unknown quality when it comes to extending such a calibrated model to situations and data it has not yet encountered. In order to be better off in such situations, calibration / validation should be used for another purpose: to help in a better understanding of these models by showing where they fall short.

This is especially important since we are entering an age with a mix of autonomous (of various degrees, even intelligent cruise control resembles an autonomous car) and human-driven vehicles, and this interaction is only poorly understood right now. Since the controller of an autonomous vehicle can be modeled with ease, the human drivers are a more challenging modeling endeavor. And it is important there to have the correct model, since so far we do not have good data that can be used to calibrate models that deal with this interaction.

Interestingly, as has already been mentioned in [3], performing a good calibration almost eliminates the differences between the models in terms of their ability to describe real-world phenomena. This result seems to be robust with respect to different data, different scenarios, different calibration methodologies, different objective functions, and different measures for the goodness of fit if the process is correctly suited. Note, however, that it was shown in [17] that some bad combinations (especially when the downhill simplex method is applied) can lead to bad results. Nevertheless, the homogeneity of the calibration results between the various models is surprising, and asks for an explanation.

The present text makes the hypothesis that this is due to a false treatment of stochastic models. In traffic flow there are two main types of stochastic models: car following models (CF) and lane changing models (LC). To demonstrate this in the case of CF we use a few synthetic examples. Using synthetic trajectories of cars following a synthetic driver according to a known model with given parameters was already done in [14, 15] and also in [16]. The idea of those two groups of authors was to test the complete calibration procedure to determine its ability to find the true optimal parameters of the original synthetic car-following behavior. This was done when measurement errors were added to synthetic trajectories [14, 15] and when different calibration procedure were used.

These papers tested the ability of the calibration procedure to cope with two of the various types of stochasticity. Indeed looking at the simulation procedure, one have to face three types of it: the noise in data (which generates an inaccuracy in the observation of the reality), the variability among parameters characterizing different drivers / vehicle couples and the stochasticity in the model itself. The various papers referenced above cope with stochasticity in the data (adding noise into synthetic data for example) or examine the variability of parameters.

The aim in this present study is to exemplify that when the stochasticity is inside the model itself, then the calibration procedure can fail to reproduce the real parameters even with a correct calibration procedure and no added noise to the speed measurements of the synthetic trajectory. It demonstrates that under not too exotic conditions the parameter estimation of the calibration process can yield results that have nothing to do with the real parameters present, while still yielding a reasonable fit. This result is rather similar to what was already observed in the case of the stochastic LC model in [10]. So far, no remedy is known for work around such a result.

The present paper is organized as follows: Section 2 presents the characteristics of an example data set with special focus on the noise in empirical data. In contrast, section 3 considers the noise term in stochastic car-following models themselves. Subsequently, the parameter estimation of four variants of a simple stochastic car-following model to synthetic trajectory data is analyzed in section 4. Finally, section 5 concludes the results of this paper.

2 STOCHASTICITY IN DATA

When models face reality, a number of issues have to be considered. All data contain experimental noise to a certain extent. To exemplify this, the paper considers data from a large German project named simTD (Safe and Intelligent Mobility Test Field Germany [1]), which aimed at a better understanding of communication in traffic and sported a fleet of equipped vehicles that drove around in the Frankfurt region for several months. Please take in mind that all data collection processes, especially when dealing with trajectory data collection, result in noisy data.

The data in the simTD project have been sampled from August to December 2012 (on 97 days) from a fleet of 125 vehicles that were driven by a few hundred different drivers. Each vehicle was equipped with at least an acceleration sensor and a speed sensor, most of the vehicles also had equipment to monitor the distance and speed-difference to a lead vehicle. In addition, they had communication devices on board and monitored their position via GPS, and vast amounts of data from their CAN bus. All the data, including the communication protocol and many more had been recorded by computers in the vehicles and subsequently transferred to a common data-base. Note, however, that especially the distances and speed-differences have been recorded by an equipment that is also in use for the driver assistance system in those cars. There were no special measuring instruments designed for scientific experiments.

From this massive data-base (which contains in zipped format 1.3 TByte of data) a few examples have been picked to be presented here. E. g., in Figure 1 the raw data from the acceleration sensor is displayed. Since a vehicle is a heavy object, its acceleration versus time curve should be fairly smooth. However, as

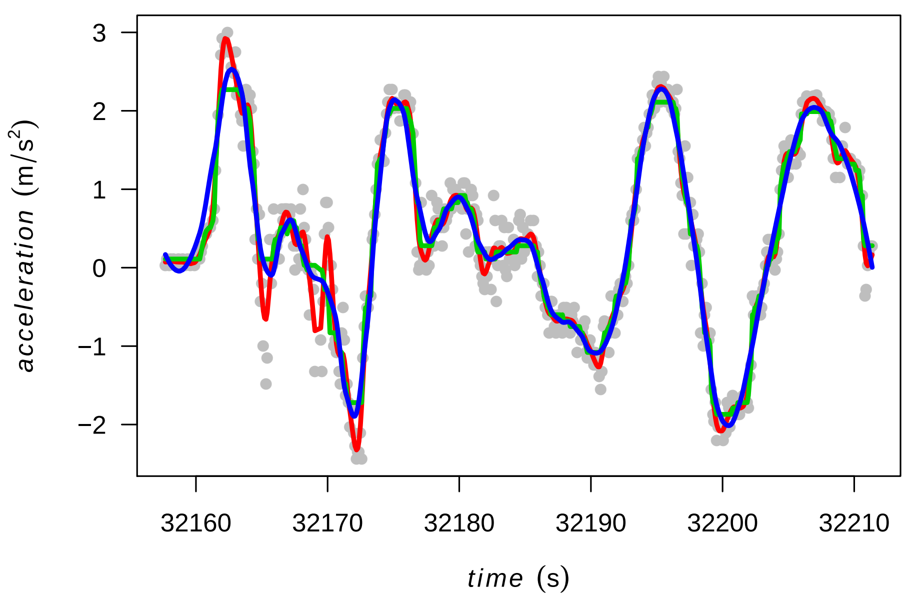

FIGURE 1 A short piece of acceleration as function of time for the vehicle 501 from the simTD data-base on August 27, 2012. The gray points are the raw data. The three curves represent different smoothing methods: the red curve is a spline smoothing, the green one from a median filter, and the blue curve is local polynomial regression of order two to the data. All these methods are implemented in “gnu R” [18].

can be seen from the raw data, there is a considerable amount of noise even in the acceleration raw data (gray points in Figure 1).

In Figure 1, different methods for smoothing the acceleration data from this data-base have been compared. Although they yield smoothed approximations to the raw data, they have the disadvantage of many smoothing method: they make assumptions about the underlying true process, and if one of these assumptions is wrong, they fail. Nevertheless, from the different smoothed curves it can be concluded that the empirical noise in these data is between 0.1 and $0.3\,\mathrm{m/s^2}$. These numbers have been obtained by computing the root-mean-square (r.m.s.) distance between the raw data and the smoothed approximations. The value of 0.1 is for the spline interpolation, which follows the data quite closely, while the value of 0.3 is for the local polynomial regression, which in the eyes of these authors is the more reasonable choice.

To sum up, this section exemplifies that the measurement process produces intrinsically noisy data. The next section considers the stochasticity caused by the observation that drivers neither behave identically (this is why parameters are distributed) nor deterministically (this is why stochasticity is incorporated in the models themselves).

3 STOCHASTIC COMPONENTS IN MODELS

In addition to the noise from empirical data, the models with which traffic flow is described microscopically, can also have stochastic components. The noise can either be in the dynamic itself, or it is contained in the parameters: different drivers have different sets of parameters to describe their driving style. This is called driver heterogeneity. Even more confusingly, the parameters of one and the same driver may be subject to temporal changes, even during a short time-span.

Most of the models that have been described so far, however, are deterministic models. Indeed a model with no stochastic components in its core equations is deterministic even though its parameters are chosen randomly. To clarify what that means, the following model (which is modeled after [2]) will be considered: using $\dot{v}$ as the acceleration a of the subject vehicle with speed v and g as the net headway to the vehicle in front, its basic version reads:

$$\dot{v} = a = B\left(\omega^2 \left(g - g^*\left(v, \Delta v\right)\right)\right). \tag{1}$$

Here, the function $B()$ limits the acceleration to fall in the interval [-β, $\alpha(v)$], where $\alpha(v)$ describes how the maximum acceleration decreases with speed. For this, any model might be acceptable. To be specific $\alpha(v) = \gamma(v_{\max} - v)$ is often used, thereby introducing the parameters $v_{\max}$ and γ. See also Figure 2 for an example how the acceleration values in real data are distributed as function of speed.

The preferred distance g^* depends on the preferred time headway T and, in addition, on the speed difference $\Delta v = V - v$ between the lead vehicle's speed V and the following vehicles speed v:

$$g^*(v, \Delta v) = v\left(T + \frac{\Delta v}{b}\right). \tag{2}$$

In this equation, the parameter b is some preferred deceleration the driver typically wants to apply in a normal car-following situation. This is well different from the parameter β that limits the maximum deceleration either to the physically possible one, or to the maximum deceleration the driver applies even in a critical situation (which is typically smaller than the physical boundary of the vehicle itself).

This model has a number of more or less obvious relatives, like the Helly model [7], the Newell model [13], a cellular automaton model [12], or even a kind of brute force linearization of the Gipps and Krauß models [6, 11]. It also shares some similarity with the IDM [20].

The numbers γ, ω are constants (for each driver!). To make the units in the equation correct, in addition, they are inverse relaxation times. So, each driver is described by the set of six parameters $v_{\max}$, b, T, β, γ, ω.

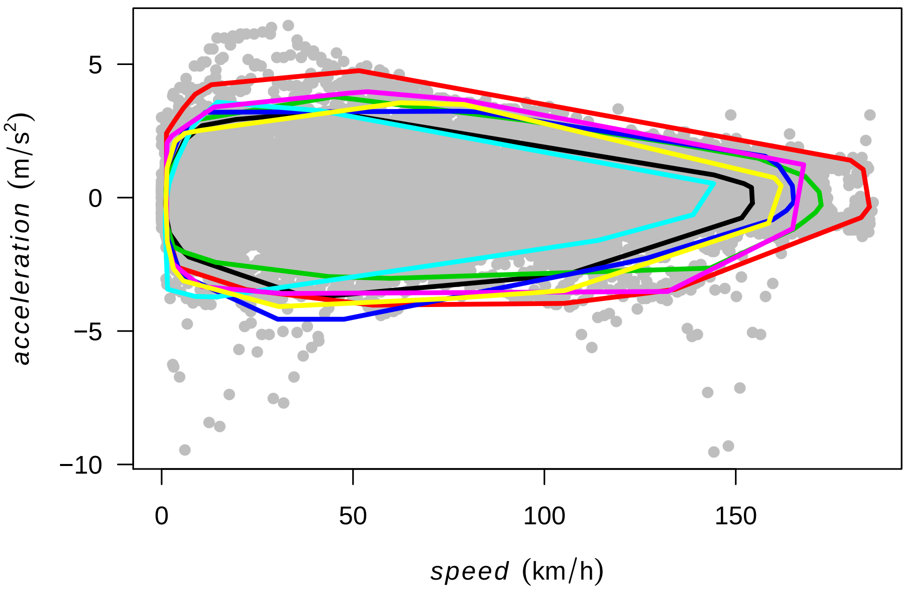

FIGURE 2 The convex hull of the acceleration and speed values of seven different drivers (colored lines) together with the raw values (gray points) of the "red" driver. The data were sampled on one day, October 22, 2012. To compute the convex hull, data-points which were hit less than 5 times have been omitted. This demonstrates that drivers occupy on average very similar regions in this (v, a)-space.

Note, that these parameters are in principle directly measurable, without getting them from a calibration process. (Clearly, the result from such a direct estimation may differ from the results of a calibration.) The acceleration bounds can be read off a time-series $a(t)$ or from Figure 2, the parameters b and ω from a plot of acceleration versus speed difference or distance, respectively, and the preferred headway T from a fit of g versus speed v. In addition to that, they can also be estimated by fitting such a model to real data, e. g. from car-following episodes.

As mentioned already, it is assumed here that each driver has its own set of parameters, and these parameters may change depending on external influences (weather, mood, level of stress etc.). However, as long as the changes are slow compared with typical time-scales in the model, they can be considered as constant and do not interfere with the dynamic. This is still a deterministic model, and it is formulated as a differential equation. This means that the driver is applying her control in each instant of time.

Adding noise to such a model leads to a stochastic differential equation:

$$\dot{v} = a = B\left(\omega^2 \left(g - g^* \left(v, \Delta v\right)\right)\right) + \sigma\xi. \tag{3}$$

The size of the noise σ is of course another parameter, while ξ stands for the noise term itself. It is important to note that the noise term should be bounded (it cannot be normally distributed) and that it must have a memory, which means that acceleration cannot change in 1 ms or so, but changes slowly, which is true for vehicles with average masses above 1000 kg. A noise term with such a memory is named colored noise.

This memory of the acceleration is an empirical feature. Real acceleration time-series have an auto-correlation function that drops from 1 for time lag 0 to $1/e$ for a time lag between two and four seconds. This can be seen in Figure 3, where the same data as in Figure 2 have been used to compute the auto-

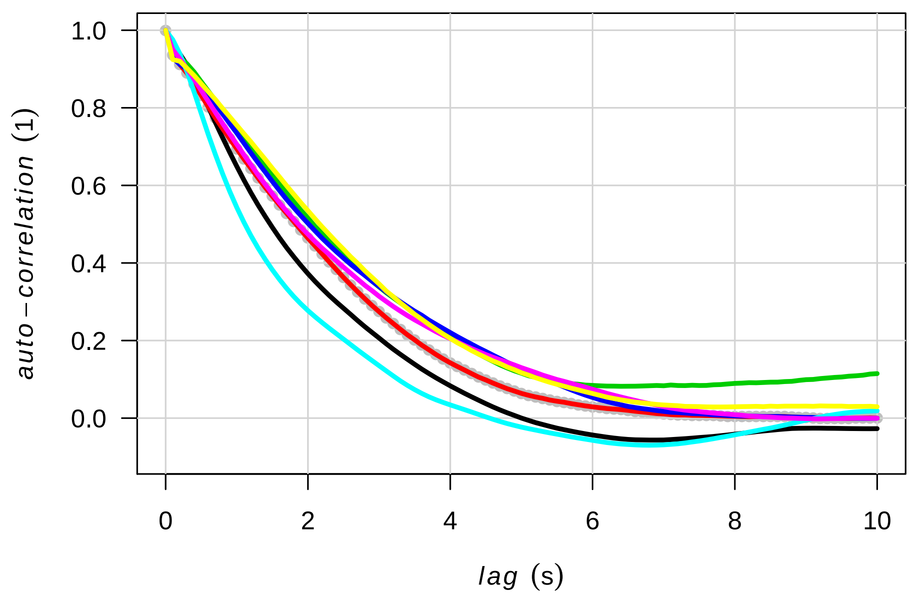

FIGURE 3 Auto-correlation function of the acceleration of seven vehicles, again taken from the simTD data.

correlation function of the seven vehicles selected from the simTD data-base, also from October 22, 2012. The auto-correlation function is computed by:

$$c(\tau) = \frac{1}{(n-\tau)\,\sigma_a^2} \sum_{0}^{n-\tau} \hat{a}(t)\,\hat{a}(t+\tau),$$

where the variable $\hat{a}(t)$ is the acceleration from which the mean value is subtracted, while σ_a is the standard deviation. Again, this is computed using “gnu R” [18]. Using pure white noise gives an acceleration time series with zero correlation time, which would not be valid under a physicist’s point of view.

A different model may be obtained by assuming that a driver does not react permanently, but only from time to time. These so called action-point models [19] have discrete points in time where acceleration (more precisely: driver’s control of it) changes quickly to a new value that might be based on equation (1). In this case, the acceleration noise is added only at these action-points and by assuming that a driver is not very good at setting the acceleration based on equation (1) but adds an error to it. In this case, the action-point mechanism introduces the memory in the acceleration, since acceleration changes only little or not at all between two subsequent action-points:

$$a_k = B\left(\omega^2\left(g(t) - g^*(v(t), \Delta v(t))\right)\right) + \sigma\xi(t) \quad t = t_k, \tag{4}$$

$$x(t) = x(t_k) + v(t_k)(t - t_k) + \frac{1}{2}a_k(t - t_k)^2 \quad t \in [t_k, t_{k+1}[\,. \tag{5}$$

Recently, a new class of models has been introduced that assume that the noise is not simply in the acceleration (it may be there, in addition), but is in one or all of the parameters describing the driver [9]. The main culprit here is the preferred headway T, but other parameters might do as well. From empirical data it is well-known that the headway distance in real traffic is a very volatile variable in a wide range of numbers, typically covering a range between 0.5 and 2 times the mean value [21]. This is quite different from the fluctuations in speed difference, which are on average around a few $\mathrm{m/s}$ compared to speeds of $20\ldots40\,\mathrm{m/s}$.

So, these models assume that the parameter T changes due to a stochastic process. The consequences of such a noise mechanism will be detailed in section 4.

4 PARAMETERS ESTIMATION IN CASE OF A STOCHASTIC MODEL

To fix ideas, the model in equation (1) is used without bounding the values of acceleration and speed, i. e. $B(\cdot)$ is the identity function (1) and is thus written:

$$\dot{v} = \omega^2 \left(g - v \left(T + \frac{\Delta v}{b} \right) \right). \tag{6}$$

This reduces the number of parameters to three; the omitted three parameters are parameters that limit the dynamics, while the remaining three parameters describe the interaction between the vehicles. Another advantage of this reduction is that the model can be rewritten as a model that is linear in the three parameters p_i (and weakly non-linear in the dynamics itself):

$$\dot{v} = p_1 g + p_2 v + p_3 v \Delta v. \tag{7}$$

A simple scenario is presented in the following. A vehicle that does not follow this model but drives its own stochastic course will be used as a leading vehicle to this model. Note, that time is discretized in chunks of 0.1 s, which is a common empirical resolution. The lead vehicle's trajectory is created by drawing acceleration values A from a Laplace distribution $p(A) \propto \exp(-|A|/a_0)$, which will be changed at random points in time whose distance is drawn randomly from the interval $[0.5, 2]$ s. This yields an acceleration trajectory $A(t)$ of the lead vehicle, from which the speed $V(t)$ of the lead vehicle is computed. In addition, it is made sure that the resulting $V(t)$ remains within two bounds $[V_1, V_2]$, an example is displayed in Figure 4. Such a trajectory looks similar to real-world speed functions, and it is important that it is not a simple function. In comparison with the synthetic data previously used, this synthetic trajectory is much more realistic (see Fig. 1 of [15]). If it were simple and mostly constant, the following vehicle's behavior in the

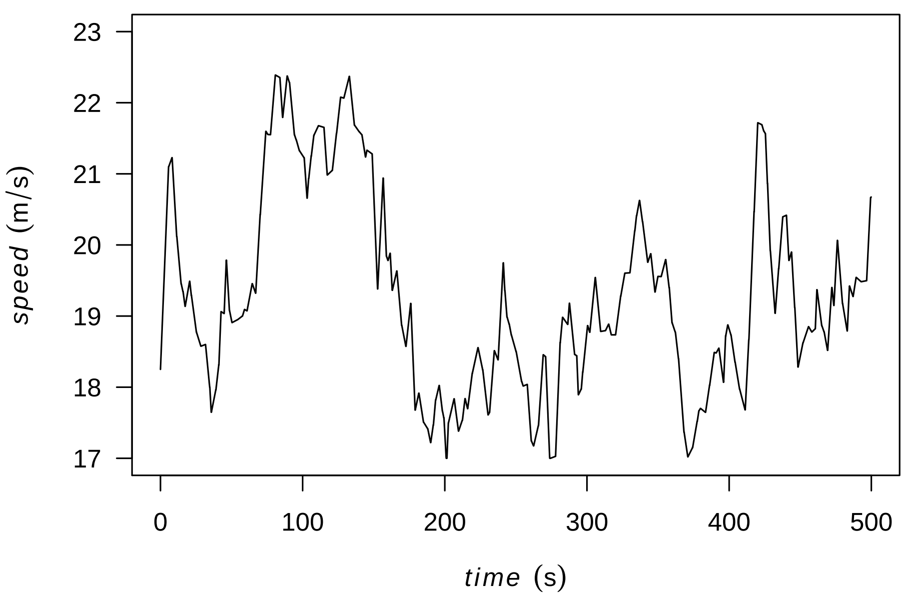

FIGURE 4 Speed versus time of a small piece of the noisy synthetic trajectory of the lead vehicle. The trajectory is bounded between 17 and 23 m/s.

case of a deterministic model would be very simple, too, and markedly different from all empirical data we have analyzed so far, which always appear very noisy. Note, however, that this noisy look of this curve is only due to the random jumps at the action-points, in between the trajectory follows a simple linear function $V(t) = V_i + a_i(t - t_i) \quad t \in [t_i, t_{i+1}[$.

The subject vehicle follows this trajectory with a certain set of parameters, and this subject vehicle is described by the model equation (7) and endowed with different noise mechanisms. In the following, the triple $p = (0.0169, -0.0239, 0.0172)$ will be used for equation (7), which has been obtained from a real-world data-set by the calibration method specified below. Note, that this set corresponds to $T = 1.41\,\mathrm{s}$ and $b = 0.98\,\mathrm{m/s^2}$, which seems a realistic choice of parameters. Running the model equation (7) with a trajectory generated for the lead vehicle gives a certain simulated trajectory $(g(t), v(t), a(t))$ of the following vehicle.

By fitting this trajectory to the model equation (7) as a simple (robust) linear fit [18], the parameters can be reproduced with a small error, which can be found in the column labeled "$\mathrm{rms}(acc, fit)$" in Table 1. This remains also true if the fitted parameters are used to run the model once more and then compare the acceleration gotten by this with the acceleration generated during the first run of the model. The corresponding values can be found in the column of Table 1 labeled by "$\mathrm{rms}(acc, sim)$". All these results are in the second row of Table 1, which is labeled "raw".

So, when the model equation (7) is following a noisy lead vehicle with speed $V(t)$, it is possible to find from the time-series the parameters that have gone into the model. This was the case for a deterministic model, which was driven by a stochastic lead vehicle. It becomes way more difficult when the stochastic variants of the model are being used. In total, the four different stochastic models have been used:

Model-1: Adding a white-noise term $\sigma\xi$ to equation (7), which could be named the physicist's approach since it is lend from the modeling of the Brownian motion.

Model-1a: Just like model 1, but instead of white noise colored noise has been added to the acceleration time-series [5, 8]. Colored noise can be understood as an exponentially smoothed white noise process, the simplest approach that has been used here is $n(t + \Delta t) = \alpha\, n(t) + (1 - \alpha)\,\sigma\xi(t)$.

Model-2: An action-point type algorithm in the acceleration, which is given by equations (4) and (5).

Model-3: In the so called 2D models [9, 21], a parameter could be changed randomly instead of fiddling around with the dynamics. The method here uses a mechanism that changes the parameter p_2 from time to time to a value that is drawn from a symmetric interval around the true value.

TABLE 1 Results of the parameter estimation for the four models.

	p_1	p_2	p_3	$\mathrm{rms}(acc, fit)$	$\mathrm{rms}(acc, sim)$
input	0.0169	-0.0239	0.0172		
raw	0.0183	-0.0259	0.0171	0.0064	0.0009
model 1	0.0170	-0.0241	0.0142	0.2981	0.4131
model 1a	0.0120	-0.0174	0.0005	0.5101	0.6079
model 2	0.0196	-0.0291	0.0158	0.1260	0.1244
model 3	0.0104	-0.0147	0.0109	0.1334	0.1416

One realization of the three models (model 1, model 2, and model 3) is displayed in Figure 5. Apart from the raw model, which has no stochastic component and is almost coincident with the input data, the different

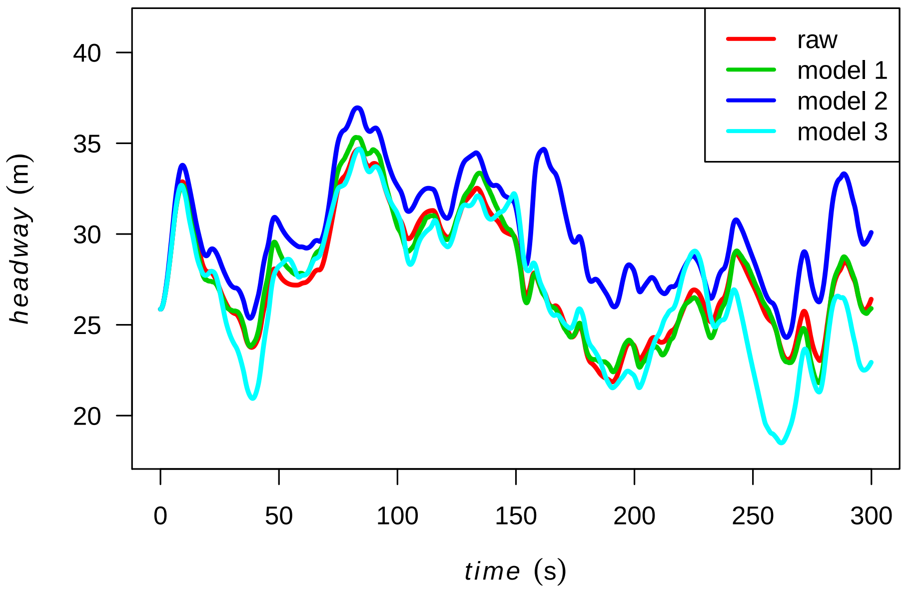

FIGURE 5 Plot of the headway versus time for the three different models, for the same lead vehicle speed function. The red curve is the original (simulated) gap that the models 1 to 3 ought to reproduce.

models lead to distance-versus-time curves. Albeit these curves do not look too different, the parameter estimation of all these stochastic models fails. The parameters are way off, and the r.m.s. error for the acceleration becomes considerable (two orders of magnitude bigger than the one obtained for the raw data!). Obviously, it depends on the parameters chosen for the size of the noise, so no general statement about its size can be made. All the results can be found Table 1.

To see that this is not just the effect from a single simulation, 100 realizations of the process have been created with the same fixed parameter set for model 3. The fitting of this simulation data to the model equation then yields a different parameter set, whose distribution is displayed in Figure 6 along with the input parameters (vertical arrows). The distribution is robust against changes in the lead vehicle's speed, i. e. different realizations of $V(t)$ give the same distribution of fitted parameters.

5 CONCLUSIONS

The literature is vast about testing calibration procedures and using simulation results to face noise in data or behaviors variability among drivers. In both cases, the last decade produced a common agreement on the solutions:

- To cope with noisy data, the simulation result is to be taken accompanied with an error estimation.
- To cope with the distribution of parameters, the simulations are repeated with random draws of the parameter set, which result in a calibrated distribution.

On the contrary, literature is scarce about calibration procedure when the stochasticity is embedded in the model itself.

This work demonstrates that most stochastic processes are potent enough to let a parameter estimation process go awry. It is not only that the goodness of fit becomes worse, in addition, even the parameters do not come out correctly. In order to minimize numerical problems, the model and the fitting procedure have

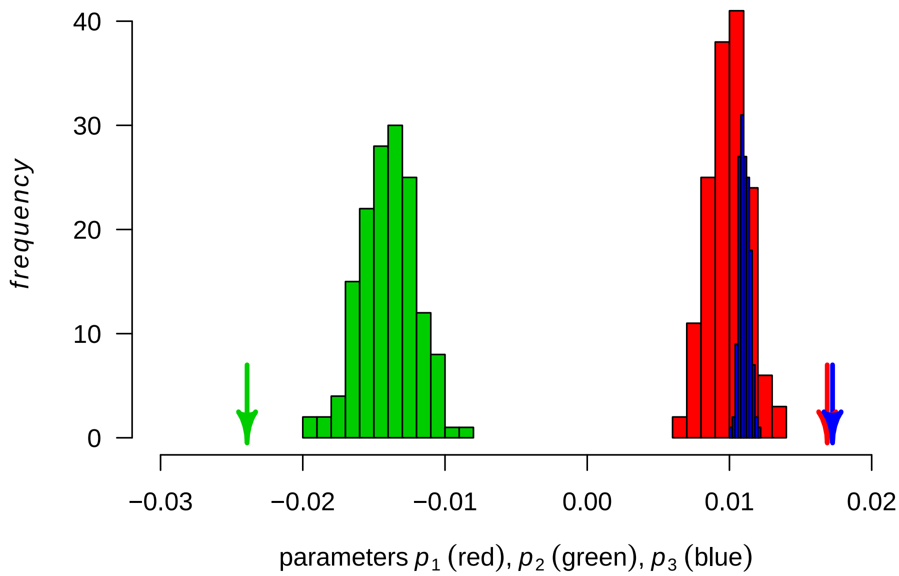

FIGURE 6 Distribution of the parameters (for model 3) estimated from 100 different runs of creating a following trajectory with the same set of parameters and then fitting them to the model. The input parameters are indicated by the three arrows pointing to the *x*-axis.

been simplified strongly so that a linear fit was sufficient, with the full statistical power that such a method provides (there are no false minima to be approached by this method, a linear fit also yields the optimum solution). In addition, all the statistical quality measures like t-values of the parameters and their respective significance levels displayed strong values indicating a really good fit nevertheless.

The important point here is that the bad fits do not manifest themselves. So, the researcher would be convinced that the parameter estimation has led to a good result. However, all of the noise models used here had the power to let the estimated parameter values come out wrongly. So far, we do not have a remedy for this. The statistics, as well as the r.m.s. and even the visual inspection of the results look quite good, but from the numerical experiments it could be seen that when fitting this type of models, the parameters cannot be estimated correctly.

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